بسمه‌تعالی

Finite Element Method

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Homework #6	Method of Weighted Residuals	

1. The following differential equation represents "One-dimensional heat conduction with linearly varying internal heat generation".

   $$k_x A \frac{d^2 T}{dx^2} + Q_0 A x = 0$$

   where k_x, Q_0, and A are constants.

   Formulate the finite element equations (that is, determine the stiffness matrix and load vectors) using Galerkin's finite element method for a two-node element of length L with the interpolation functions:

   $$N_1(x) = 1 - \frac{x}{L} \qquad N_2(x) = \frac{x}{L}$$

   Ref: Fundamentals of Finite Element Analysis, David V. Hutton, 2004.

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2. For the bar subjected to the uniform line load p in the axial direction shown in the figure, determine the displacements function using Galerkin's method. Choose a quadratic polynomial for displacement in the method as follows:

   $$u(x) = c_1 x + c_2 x^2$$

Ref: A First Course in the Finite Element Method, D. L. Logan, 5th Edition, 2011.

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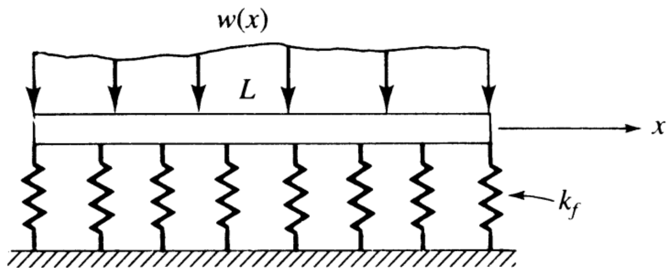
3. Derive the equations for the beam element on an elastic foundation (see Figure) using Galerkin's method. The basic differential equation for the beam on an elastic foundation is:

$$(EIv'')'' = -w + k_f v$$



Ref: A First Course in the Finite Element Method, D. L. Logan, 5th Edition, 2011.

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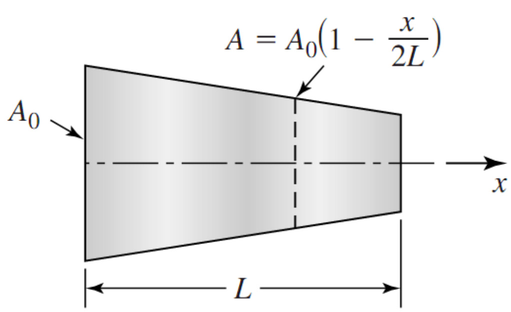
4. Consider a tapered uniaxial tension-compression member subjected to an axial load as shown in the figure.
   The cross-sectional area varies as $A = A_0(1 - x/2L)$, where L is the length of the member and A0 is the area at x = 0.
   Extract the governing differential equation for the axial displacement (u).



Ref: Fundamentals of Finite Element Analysis, David V. Hutton, 2004. (with some changes)
